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Barry Falk
Iowa State University

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Abstract

Sims (1991) conjectured that the conditional maximum likelihood estimator of trend stationary models of macroeconomic time series will tend to place initial observations relatively far from the estimated trend line. This can be misleading if the entire sample has been generated by the same data generating process and there is nothing unusual about the initial observations. We use the extended Nelson Plosser data set to evaluate Sims's conjecture. We consider the weighted symmetric estimator developed by Park and Fuller (1993) as an alternative approach to this estimation problem.

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Estimating Trend Stationary Models of Homogeneously Generated Samples

Barry Falk
Department of Economics
Iowa State University
Ames, IA 50011-1070

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Phone: (515)294-5875
Fax: (515)294-0221
Email: bfalk@iastate.edu

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ABSTRACT

Sims (1991) conjectured that the conditional maximum likelihood estimator of trend stationary models of macroeconomic time series will tend to place initial observations relatively far from the estimated trend line. This can be misleading if the entire sample has been generated by the same data generating process and there is nothing unusual about the initial observations. We use the extended Nelson-Plosser data set to evaluate Sims's conjecture. We consider the weighted symmetric estimator developed by Park and Fuller (1993) as an alternative approach to this estimation problem.

1. Introduction

This paper is concerned with estimation procedures for trend-stationary (TS) representations of macroeconomic time series that properly account for the evidence about parameters contained in the initial observations of the sample. Consequently, an assumption maintained throughout the paper is that the time series we will be working with (i.e, the extended Nelson-Plosser series) are realizations of TS processes. This assumption leans against the prevailing wind in applied macroeconomics where difference stationary representations seem to be preferred. However, there remain important pockets of skepticism about the plausibility and wisdom of the unit root approach.[1/]

The conventional approach to estimating the parameters of a TS process is to apply the maximum likelihood estimator for a Gaussian process conditional on initial observations (or to apply an asymptotically equivalent two-step estimator). This reduces to an OLS procedure and so it is much more convenient than solving the constrained numerical optimization problem implied by the unconditional maximum likelihood estimator. Furthermore, when sample size is large and the maximum autoregressive root is small relative to one, the conditional MLE will provide a good approximation to the unconditional MLE. General concerns have been raised, however, about the performance of the conditional MLE of TS models for sample sizes common in macroeconomics when the stationary component has a relatively large maximum root.[2/]

This paper will consider the prevalence of a specific flaw in TS models of macroeconomic data fit by the conditional MLE. The flaw, whose prevalence was conjectured by Sims (1991), is a propensity for the estimator to place initial observations relatively far from the estimated trend line. This tendency arises because the conditional likelihood function imposes no penalty for the location of initial observations and so the conditional MLE uses these observations as leverage in fitting the remainder of the sample.

Several adverse implications of this hypothesized feature of the conditional MLE follow. First, conditional ML estimates of TS models will tend to erroneously imply that a large part of the early sample behavior of a time series is movement from an extreme initial value back toward the long-run path (as opposed to normal variation around that path). Second, flawed estimates will generate spurious cyclical behavior in the time series contributing to erroneous inferences about the cyclical component of the series. Third, they can lead to inefficient forecasts.

The remainder of the paper is organized as follows. Section 2 will review the evidence presented by Sims (1991) that led to his conjectures regarding the finite sample performance of the conditional MLE of TS models of macroeconomic time series. Section 3 applies the conditional MLE to TS models of the extended Nelson-Plosser data series to evaluate Sims's conjectures. In Section 4 we re-estimate the TS models of the Nelson-Plosser series using a computationally convenient approximation to the unconditional MLE: the weighted symmetric (WS) estimator developed by Park and Fuller (1993). Out-of-sample forecasts of the models estimated by the conditional MLE and the WS estimator are compared in Section 5. A summary and conclusions are presented in Section 6.

2. Background

Assume that the observed time series $Y_1, \ldots, Y_T$ is part of a realization of the TS process

$$Y_t = a + bt + y_t , \qquad t = 1, 2, \ldots \tag{1}$$

where

$$\gamma(L) y_t = \epsilon_t , \qquad t = 0, \pm 1, \pm 2, \ldots \tag{2}$$

ϵ_t is a white-noise Gaussian process with constant and finite variance, σ_e^2. The roots of the complex-valued, p-th order polynomial $\gamma(z) = 1 - \gamma_1 z - \ldots - \gamma_p z^p$ are assumed to be greater than one in modulus. This model implies that

$$Y_t = \alpha + \beta + \gamma_1 Y_{t-1} + \ldots + \gamma_p Y_{t-p} + \epsilon_t \ , \ t = p+1, p+2, \ldots \tag{3}$$

where $\alpha = a(1-\gamma_1-\ldots-\gamma_p)+b(\gamma_1+2\gamma_2+\ldots+p\gamma_p)$ and $\beta = b(1-\gamma_1-\ldots-\gamma_p)$.

The maximum likelihood estimator of $\alpha, \beta, \gamma_1, \ldots, \gamma_p$ conditional on $Y_1, \ldots, Y_p$ is the OLS estimator of (3) and the conditional maximum likelihood estimates of a and b follow directly from the relationship among the parameters of (1)-(3). The strong asymptotic properties of this estimator, which do not depend on normality, are well-known.[3/] The conditional MLE is often approximated by the asymptotically equivalent two-step estimator: estimate a and b by a regression of Y_t on [1,t], t = 1,...,T and then fit the regression residuals to an AR(p) by OLS for t = p+1,...,T to estimate $\gamma_1, \ldots, \gamma_p$. That is, estimate a and b by a regression of Y_t on [1,t], t = 1,...,T and then estimate (2) by the conditional MLE with $\hat{y}_1, \ldots, \hat{y}_T$ in place of $y_1, \ldots, y_T$.

Sims (1991) used the conditional MLE to fit model (1)-(3) with p = 1 to quarterly, seasonally-adjusted, logged U.S. real GNP over the 1947:I-1991:IV sample period. He reported the following estimates:

$$Y_t = .2320 + .0002t + .9684 Y_{t-1} + \epsilon_t \ , \ \sigma_\epsilon = .0105$$

or, equivalently,

$$Y_t = 7.1204 + .0069t + y_t, \ \sigma_y = .0420$$

where

$$y_t = .9684 y_{t-1} + \epsilon_t \ , \ \sigma_\epsilon = .0105.$$

These results are illustrated in Figure 1 which includes the actual sample path, the estimated trend path, and, for reference purposes, a two-standard-error band centered on the estimated trend path. The most striking feature of this figure for our purposes is that the first half of the sample period is dominated by a large transient as real GNP moves from its extreme initial value, which is nearly four standard errors from the steady-state path, toward the steady-state path around which GNP fluctuates for the remainder of the sample period.

Extreme economic conditions around 1947:I could offer one explanation for the relationships illustrated in Figure 1. Another possible explanation is that in and before 1947:I real GNP was generated by a different process than the process generating subsequent observations. Sims's (1991) explantion is that this figure illustrates a spurious phenomenna that is likely to arise routinely when the conditional MLE is applied to estimate TS processes.

One way to evaluate Sims's explanation is to apply the conditional MLE to other data series whose initial observations occur at different points in time. If the kind of result observed in Figure 1 persists across time series and sample starting dates then this would lend credence to Sims's conjecture that the result is an artifact attributable to the conditioning aspect of the estimation procedure rather than an accurate reflection of the underlying data generating process.

3. Conditional ML Estimates of TS Models of the Nelson-Plosser Data

We will examine the extended Nelson-Plosser data set, which was initially used by Nelson and Plosser (1982) and was updated by Schotman and van Dijk (1991). These data are are frequently used to evaluate estimation and test procedures being used or being proposed for use to study the time series properties of macroeconomic data. The 14 annual time series that make up this data set and their sample periods are: real

GNP (1909-1988), nominal GNP (1909-1988), GNP deflator (1889-1988), real wages (1900-1988), nominal wages (1900-1988), employment (1890-1988), unemployment rate (1890-1988), industrial production (1860-1988), nominal money stock (1889-1988), velocity (1869-1988), nominal interest rate (1900-1988), consumer prices (1860-1988), and stock prices (1871-1988).

Conditional ML estimates of the parameters in (1)-(3) were obtained for each of the 14 series in the extended Nelson-Plosser data set. We set p = 3 in each case, sidestepping the issue of optimal lag length selection. Because the velocity and consumer price series clearly fit the quadratic trend version of the model much better than they fit the linear trend version, we report the quadratic trend estimates for these two series.[4/]

The conditional ML estimates of (1)-(3) are summarized in Table 1 along with some statistics we will discuss below. According to this table the conditional ML estimates of the AR components satisfy the stationarity condition in all 14 cases. The maximum AR root ranges from .643 (velocity) to .951 (interest rate). The relatively large sizes of these maximum roots signal that the conditional MLE may not be a good approximation to the unconditional MLE.

The summary statistics presented in Table 1 indicate that there is a tendency for the conditional MLE to place initial values of each series relatively far from the estimated trend line regardless of the starting date. In most cases, however, the situation is not nearly as extreme as Sims (1991) found in his study of post-war, quarterly, real GNP. Notice that in 11 of the 14 cases the absolute value of $\hat{y}_1$ is larger, usually much larger, than the absolute value of $\hat{y}_T$. In 4 of the 14 cases $\hat{y}_1$ is more than two standard errors from zero, in 9 of the 14 cases it is more than one standard error zero, and in 12 of the 14 cases it is more than

.8 standard errors from the zero. In contrast, $\hat{\gamma}_T$ is never more than two standard errors from zero, it is more than one standard error from zero in only 3 of the 14 cases, and it is more than .8 standard errors from zero in only 6 of the 14 cases.

Figures 2.1 - 2.14 illustrate each series, its estimated trend, and, for reference purposes, a two-standard-error band centered on the estimated trend line. The figures provide visual confirmation of the preceding remarks regarding the statistics reported in Table 1. They also illustrate that a two-standard-error band around the estimated trend line is usually sufficient to contain most of the actual time path of a series except for the first part of the sample. Finally, they indicate that a large part of the early sample behavior of industrial production, nominal GNP, nominal wages, the nominal interest rate, stock prices, and the GNP deflator appear to be driven by movements from extreme initial conditions back toward their respective long-run paths.

In summary, the results we have presented in this section appear to support Sims's (1991) conjecture that conditional ML estimates of TS macroeconomic time series tend to place initial observations relatively far the estimated trend line. This occurs in most of the series we have considered in spite of the differences among their starting dates. When it occurs it can easily imply quite different behavior for the series in the first part of the sample than the behavior implied for the latter part of the sample.

4. WS Estimates of TS Models of the Nelson-Plosser Data

One alternative to the conditional MLE of (1)-(3) is the unconditional MLE based on the unconditional stationary distribution of $y_1, \ldots, y_p$. This is a constrained (by the stationarity condition) nonlinear optimization problem that must be solved numerically. We propose the use of the weighted symmetric (WS) estimator developed by Park and Fuller (1993) as a computationally convenient approximation to the MLE in settings such as ours.[5/]

In this scetion of the paper we apply the WS estimator to fit the extended Nelson-Plosser data to the TS model. We begin with a brief description of the WS estimator.[6/] Assume that y_t is the stationary process described by (2). Then y_t also has a backward-evolving representation

$$y_t = \gamma_1 y_{t+1} + \ldots + \gamma_p y_{t+p} + v_t \ , \quad t = 0, \pm 1, \pm 2, \ldots \tag{4}$$

where v_t is a white noise process with variance σ_ϵ^2. Note that the parameters in (2) and (4) are equivalent. Given sample data $y_1, \ldots, y_T$, models (2) and (4) suggest a class of estimators of the form: Choose $\gamma_1, \ldots, \gamma_p$ to minimize $Q(\gamma)$ where

$$Q(\gamma) = \sum_{t=p+1}^{T} w_t (y_t - \gamma_1 y_{t-1} - \ldots - \gamma_p y_{t-p})^2 + \sum_{t=1}^{T-p} (1-w_{t+1})(y_t - \gamma_1 y_{t+1} - \ldots - \gamma_p y_{t+p})^2 \tag{5}$$

and $w_1, \ldots, w_T$ is a given set of weights. The OLS estimator is the member of this class obtained by setting $w_t = 1$ for $t = 1, \ldots, T$. The simple symmetric estimator described by Dickey, Hasza, and Fuller (1984) is obtained by setting $w_t = 0.5$, $t = 1, \ldots, T$. The WS estimator is obtained by setting w_t as follows:

$$\begin{aligned} w_t &= 0 && t = 1,2,\ldots,p \\ &= (t-p)/(T-2p+2) && t = p+1,\ldots,T-p+1 \\ &= 1 && t = T-p+2,\ldots,T \end{aligned} \tag{6}$$

if $p > 1$. If $p = 1$, set $w_t = t-1/T$ for $t = 1, \ldots, T$.

The solution to (5) for any set of weights is simply

$$\gamma = (X'WX)^{-1} X'WZ \tag{7}$$

where

W is a (2T-2p)x(2T-2p) diagonal matrix with

$W_{ii} = w_{p+i}$ and $W_{T-p+i,T-p+i} = 1 - w_{i+1}$, $i = 1,\ldots,T-p$;

Z is a (2T-2p)x1 column vector with

$Z_i = y_{p+i}$ and $Z_{T-p+i} = y_i$, $i = 1,\ldots,T-p$;

and

X is a (2T-2p)xp matrix with

$X_{ij} = y_{p+i-j}$ and $X_{T-p+i,j} = y_{i+j}$, $i = 1,\ldots,T-p$.

Thus, the WS estimator of γ, $\hat{\gamma}_{ws}$, is calculated from (7) where the weighting matrix W is specified using the weights defined in (6).

To apply the WS estimator to estimate the TS model (1)-(3), we replace y_t in (5) with Y_t - a - bt and minimize Q with respect to γ, a, and b. The solution to this problem is nonlinear. Pantula, et al [1994] suggest approximating the solution by applying OLS to (1) to estimate a and b and then using the OLS residuals from (1) in place of y_t to estimate γ. We will apply the following iterative linear procedure which we have found yields approximately the same solution as the numerically-derived solution, typically in just a few steps.

1. Obtain initial estimates of a and b (by OLS or the conditional MLE).

2. Use the estimates of a and b to construct estimates of $y_1,\ldots,y_T$ from (1) then apply the WS estimator to estimate $\gamma(L)$.

3. Apply the WS estimator to estimate a and b conditional on $\gamma(L)$.

4. Repeat step 2.

The results from the application of this procedure to the extended Nelson-Plosser data are summarized in Table 2. Consumer prices and velocity were fit to models with a quadratic trend. According to Table 2, the stationarity condition is satisfied in all 14 cases with the maximum AR root ranging from .646 (velocity) to .943 (interest rate).

The summary statistics presented in Table 2 indicate that the WS estimator does take care of the problem we highlighted with the conditional ML estimator. This is not surprising since the WS estimator equally weights initial and final observations. In about half of the cases (6 out of 14) the absolute value of $\hat{y}_1$ is larger than $\hat{y}_T$. Neither $\hat{y}_1$ nor $\hat{y}_T$ is ever more than two standard errors from zero.[7/] $\hat{y}_1$ is more than one standard error from zero in 4 out of 14 cases and $\hat{y}_T$ is more than one standard error from zero in 5 out of 14 cases.

It is clear from Figures 2.1-2.14 and Table 2 how the WS estimator adjusts the conditional ML estimates. In cases where the conditional MLE places initial observations far above (below) the estimated trend path, e.g., real GNP, the WS estimator increases (decreases) the intercept and decreases (increases) the slope of the trend line.

The main conclusion to be drawn from this section is that the WS estimator provides a computationally convenient alternative to the unconditional MLE that, in contrast to the conditional MLE, tends to provide a homogeneous explanation of the entire sample.

5. Forecast Comparisons

The WS estimator provides a more homogeneous explanation of the entire sample than does the conditional MLE. This is not necessarily a desirable feature of the WS estimator when there is reason to believe that initial observations were not generated from the stationary distribution implied by the model this is not necessarily a desirable feature of the WS estimator. From this perspective the choice between the WS estimator and the conditional MLE needs to be considered on a case-by-case basis in applied work. The conditional MLE could also be preferable to the WS estimator if there is some nonlinearity in the true data generating process and the conditional MLE of the linear model is conveniently picking this up by its extreme placement of initial observations. We consider this possibility for the extended Nelson-Plosser data series by comparing out-of-sample forecasts generated by models fit by each of the two estimators.

The 14 series in the extended Nelson-Plosser data set differ in their starting dates but the last observation is always 1988. We refit model (1)-(3) by the conditional ML and the WS estimators with $p = 3$ and T corresponding to 1978. (As before, consumer prices and velocity were fit to a quadratic-trend version of (1)-(3).) Using the re-estimated models and the observed values of the data through 1978, forecasts were generated for 1979-1988. That is, s-step ahead forecasts were generated for $s = 1,\ldots,10$ based on models fit to truncated samples. Table 3 reports the root mean squared errors (RMSE's) of these forecasts for forecast horizons 1, 2, 5 and 10.

According to the results of this exercise, the relatively large weight that the conditional MLE places on later observations in a sample does not necessarily translate into superior short- to medium-term forecast performance. In eight out of 14 cases, the RMSE's corresponding to forecasts generated by models fit by the WS estimator are uniformly less than the RMSE's corresponding to forecasts generated by models fit by the conditional ML estimator. The conditional ML estimator uniformly dominates the WS estimator in this exercise for only five cases.

6. Conclusion

This paper has been concerned with the estimation of trend stationary models of macroeconomic time series. A study of the extended Nelson-Plosser data set seems to confirm Sims's (1991) conjecture that the conditional maximum likelihood estimator of trend stationary models will tend to place the initial observations upon which the estimator is conditioned relatively far from the estimated trend line. The conditional MLE also displays a tendency to imply that much of the early sample behavior of a series is transient behavior as the series moves from its extreme initial position back toward the trend path. These findings suggest that the conditional MLE may be inappropriate in situations where there is no compelling reason to believe that the initial observations are unusual in terms of the process generating the remainder of the sample.

The extended Nelson-Plosser data series were re-fit to trend stationary models using the weighted symmetric estimator developed by Park and Fuller (1993). The WS estimator is an approximation to the unconditional MLE, though it is much easier to compute. Estimates of the trend stationary model using the WS estimator appear to provide a homogeneous explanation of the entire sample. The estimated models do not tend to be characterized by extreme initial conditions or early sample transient behavior. Furthermore, the estimator's symmetric treatment of initial and terminal observations does not seem to come at the expense of the fitted model's out-of-sample forecast performance.

We conclude that in those situations where a homogeneous explanation of the entire sample is preferred, the WS estimator provides an attractive alternative to the conditional MLE. Although we have focussed on univariate processes, presumably the argument extends to the multivariate case where a multivariate version of the WS estimator could be applied to estimate vector autoregressive representations of trend stationary vector processes.

NOTES

1. See, for example, Sims (1988), DeJong and Whiteman (1991), Sims and Uhlig (1991), Hamilton (1994, pp. 444-447) and the special issue of Volume Six of the *Journal of Applied Econometrics* devoted to this controversy.

2. See Davidson and MacKinnon (1993), pp. 342-351.

3. See Hamilton (1994), Chapter 16.

4. The quadratic trend version of (1)-(3) is

$$Y_t = a + bt + ct^2 + y_t , \quad t = 1,2,... \tag{1'}$$

$$\gamma(L)y_t = \epsilon_t \quad , \quad 0,\underline{+}1,\underline{+}2,... \tag{2'}$$

and,

$$\gamma(L)Y_t = \alpha + \beta t + \delta t^2 + \epsilon_t \quad , \; t = p+1,\; p+2,\; ... \tag{3'}$$

where $\alpha = a\gamma(1) + b(\gamma_1 + 2\gamma_2 + ... + p\gamma_p) - c(\gamma_1 + 4\gamma_2 + ... + p^2\gamma_p)$; $\beta = b\gamma(1) + 2c(\gamma_1 + 2\gamma_2 + ... + p\gamma_p)$; and $\delta = c\gamma(1)$.

5. The two-step OLS approximation to the conditional MLE (i.e., estimate (1) by OLS then apply OLS to (2) using the residuals from (1) in place of y_t) is also a computationally convenient alternative that treats the first and last observations symmetrically with respect to the parameters a and b. However, the initial observations problem remains with respect to the estimates of $\gamma(L)$, α, and β. Furthermore, since we are operating in an environment in which the conditional MLE may not be providing a good approximation to the unconditional MLE, the jusitfication for applying an approximation to the conditional MLE is weakened.

6. See Fuller (1992), Pantula, Gonzales-Farias, and Fuller (1994), and Park and Fuller (1993) for more details.

7. The standard errors in Table 2 were computed as follows. The WS estimates were used to construct the WS residuals $\hat{\epsilon}_{p+1}, \ldots, \hat{\epsilon}_T, \hat{v}_1, \ldots, \hat{v}_{T-p}$, where $\epsilon_t = \gamma(L)Y_t - \alpha - \beta t$ and $v_t = \gamma(L^{-1}) - \alpha - \beta t$. The sum of squared residuals was divided by 2T - 11 (=2(T-p)-(p+2)) to get an estimate of $\hat{\sigma}_\epsilon^2$. This estimate and the estimates of γ_1, γ_2, and γ_3 were used to estimate σ_y^2 based on the stationary distribution of y_t implied by (2). Notice that the standard errors of y_t are reasonably close across Tables 1 and 2 so that the comparisons being made in this section are sensible.

Table 1

Conditional ML Estimates of the TS Model

Series	a	b	c	γ_1	γ_2	γ_3	λ_{max}	σ_y^2	$\lvert\hat{y}_1/\hat{y}_T\rvert$	$\lvert\hat{y}_1/\hat{\sigma}_y\rvert$	$\lvert\hat{y}_T/\hat{\sigma}_y\rvert$
Real GNP	4.529	0.032	-	1.218	-0.370	-0.034	0.681	0.114	9.45	1.75	0.19
Real GNP Per Capita	6.923	0.020	-	1.199	-0.358	-0.038	0.680	0.112	153.6	1.97	0.01
Industrial Production	0.114	0.041	-	0.947	-0.164	0.043	0.810	0.168	2.48	1.55	0.62
Employment	10.077	0.016	-	1.272	-0.488	0.080	0.776	0.076	1.39	0.88	0.64
Unemployment Rate	1.915	-0.002	-	0.991	-0.414	0.168	0.737	0.632	17.33	0.83	0.05
Real Wages	2.899	0.017	-	1.210	-0.315	0.039	0.913	0.111	0.35	0.49	1.39
Nominal Wages	5.710	0.048	-	1.473	-0.611	0.078	0.875	0.208	3.64	2.07	0.57
Nominal GNP	9.777	0.068	-	1.440	-0.570	0.074	0.897	0.306	3.24	1.86	0.58
GNP Deflator	2.373	0.034	-	1.410	-0.384	-0.060	0.924	0.233	4.46	3.64	0.82
Money Supply	1.161	0.062	-	1.642	-0.755	0.055	0.786	0.214	0.22	0.27	1.23
Interest Rate	-0.699	0.107	-	1.170	-0.379	0.162	0.951	1.974	7.12	1.97	0.28
Stock Prices	0.196	0.042	-	1.176	-0.403	0.159	0.926	0.431	3.17	3.04	0.96
Consumer Prices	3.880	-0.024	0.0003	1.646	-0.921	0.200	0.820	0.153	11.86	3.66	0.87
Velocity	1.843	-0.035	0.0002	0.916	-0.126	-0.103	0.643	0.091	0.69	0.92	1.85

Notes: The numbers under columns a, b, c, $\gamma_1, \gamma_2, \gamma_3$, and σ_y^2 are the conditional maximum likelihood estimates of the corresponding parameters of equations (1) and (2). The parameter c enters as the coefficient on t^2 in the quadratic version of (1); λ_{max} is the maximum estimated AR root of (2); $\hat{y}_t$ is the deviation of the sample value from the fitted trend line in period t.

Table 2

Weighted Symmetric Estimates of the TS Model

Series	a	b	c	γ_1	γ_2	γ_3	λ_{max}	σ_y^2	$\lvert\hat{y}_1/\hat{y}_T\rvert$	$\lvert\hat{y}_1/\hat{\sigma}_y\rvert$	$\lvert\hat{y}_T/\hat{\sigma}_y\rvert$
Real GNP	4.591	0.031	-	1.228	-0.369	-0.033	0.681	0.121	23.2	1.15	0.05
Real GNP Per Capita	6.989	0.018	-	1.215	-0.358	-0.039	0.683	0.120	5.25	1.30	0.25
Industrial Production	0.022	0.042	-	0.958	-0.165	0.041	0.819	0.175	1.18	0.96	0.81
Employment	10.059	0.016	-	1.263	-0.482	0.080	0.773	0.076	1.24	0.65	0.53
Unemployment Rate	1.706	0.001	-	0.978	-0.402	0.170	0.744	0.643	5.29	0.50	0.09
Real Wages	2.944	0.016	-	1.198	-0.310	0.045	0.913	0.110	0.07	0.09	1.14
Nominal Wages	5.961	0.045	-	1.476	-0.604	0.069	0.878	0.218	0.96	0.84	0.88
Nominal GNP	10.132	0.063	-	1.438	-0.561	0.065	0.890	0.309	0.83	0.72	0.86
GNP Deflator	2.990	0.027	-	1.442	-0.391	-0.077	0.938	0.288	0.81	0.83	1.02
Money Supply	1.670	0.062	-	1.625	-0.741	0.053	0.778	0.207	0.20	0.25	1.26
Interest Rate	2.469	0.066	-	1.164	-0.371	0.154	0.943	1.889	0.79	0.40	0.51
Stock Prices	0.999	0.034	-	1.197	-0.403	0.150	0.937	0.484	0.87	1.06	1.21
Consumer Prices	3.590	-0.016	0.0003	1.678	-0.952	0.206	0.829	0.169	2.63	1.64	0.63
Velocity	1.794	-0.034	0.0002	0.928	-0.133	-0.101	0.646	0.094	0.34	0.38	1.13

Notes: The numbers under columns $a, b, c, \gamma_1, \gamma_2, \gamma_3$, and σ_y^2 are the weighted symmetric estimates of the corresponding parameters of equations (1) and (2). The parameter c enters as the coefficient on t^2 in the quadratic version of (1); λ_{max} is the maximum estimated AR root of (2); $\hat{y}_t$ is the deviation of the sample value from the fitted trend line in period t.

Table 3

Root Mean-Squared-Errors of One- to S-Step-Ahead Forecasts

Series	Conditional ML: s=1	s=2	s=5	s=10	Weighted Symmetric: s=1	s=2	s=5	s=10
Real GNP	.0125	.0329	.0727	.0672	.0066	.0226	.0509	.0387
Real GNP Per Capita	.0103	.0294	.0628	.0507	.0042	.0189	.0406	.0308
Industrial Production	.0045	.0458	.1301	.1433	.0117	.0563	.1498	.1724
Employment	.0089	.0073	.0073	.0323	.0074	.0054	.0094	.0247
Unemployment Rate	.0401	.2024	.4681	.4138	.0042	.1504	.3844	.3163
Real Wages	.0591	.1132	.1790	.2385	.0517	.1010	.1562	.2056
Nominal Wages	.0267	.0490	.1278	.1720	.0321	.0592	.1532	.2176
Nominal GNP	.0357	.0586	.1300	.2162	.0434	.0727	.1634	.2745
GNP Deflator	.0497	.0895	.2008	.2895	.0336	.0592	.1208	.1346
Money Supply	.0227	.0447	.1542	.2915	.0221	.0432	.1485	.2780
Interest Rate	.6551	1.875	2.915	2.400	.7479	2.024	3.233	2.639
Stock Prices	.0493	.1186	.2143	.4806	.0321	.0912	.1699	.4101
Consumer Prices	.0499	.1002	.1722	.1784	.0242	.0472	.0477	.1488
Velocity	.0026	.0094	.0358	.0958	.0296	.0612	.1445	.2988

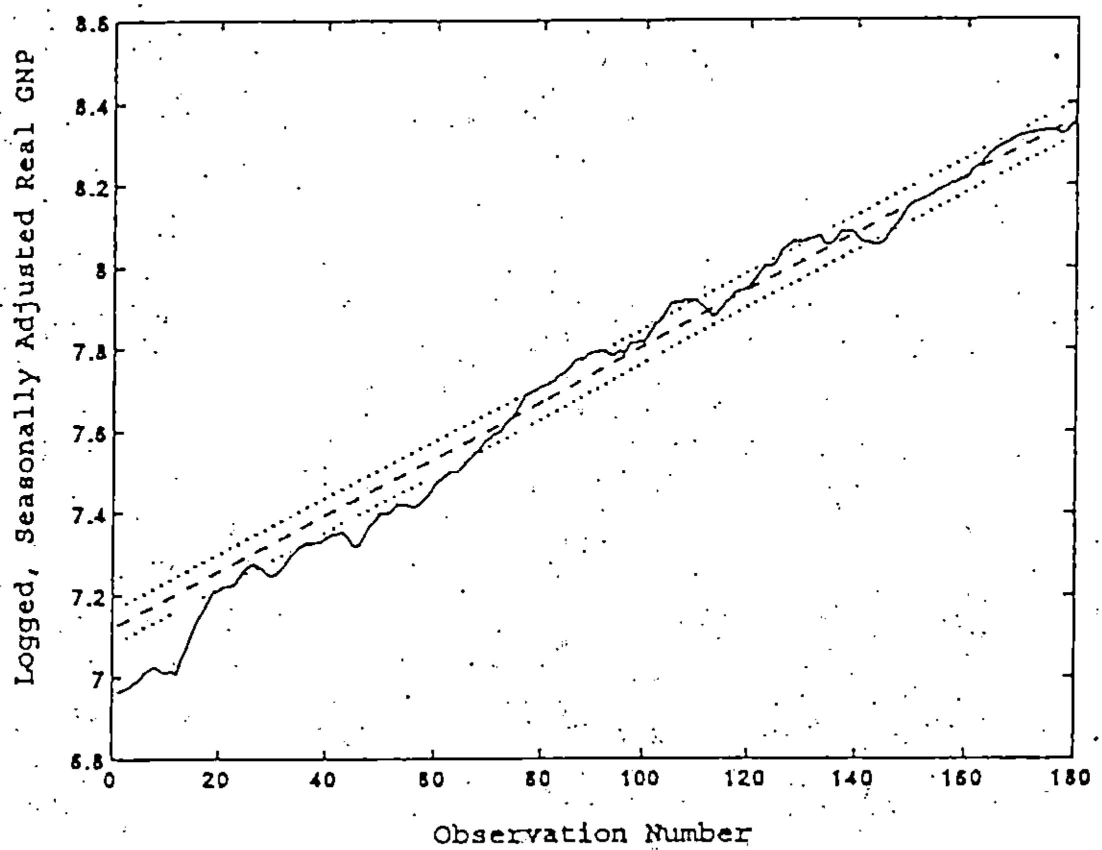
Notes: Each series was refit to model (1)-(3) with sample truncated at period T - 10 (= 1978). s-step ahead forecasts were generated for s = 1,...,10. For each s, the root MSE of forecasts for periods T-10+1,...,T-10+s was computed and these values are reported above.

Figure 1

Trend-Stationary Representation of Quarterly Real GNP: Conditional ML Estimate

____ = sample path, --- = trend path, ... = two-standard-error band

Observation 1 = 1947:I , Observation 180 = 1991:IV



Figures 2.1 - 2.14

Trend-Stationary Representations of Nelson-Plosser Series: Conditional MLE

____ = sample path , --- = trend path, ... = two-standard-error band

Figure 2.1 - Real GNP

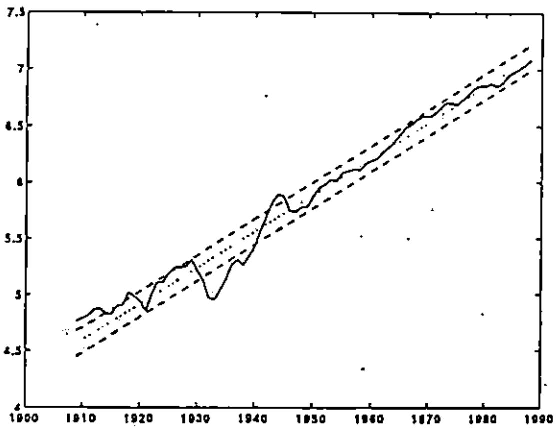


Figure 2.2 - Real GNP Per Capita

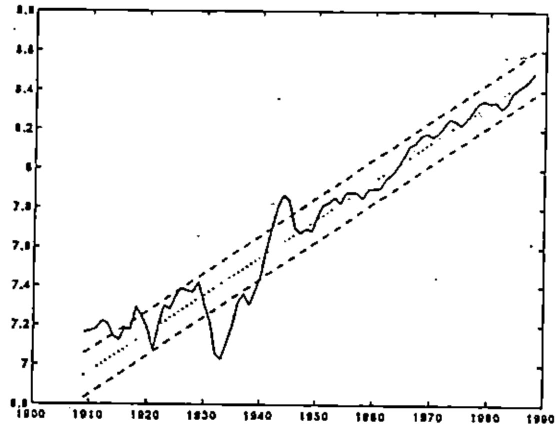


Figure 2.3 - Industrial Production

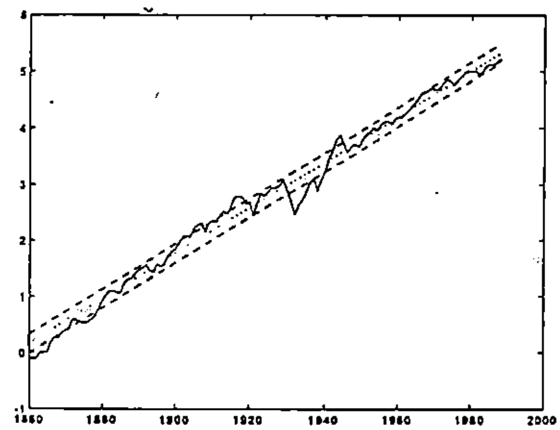


Figure 2.4 - Employment

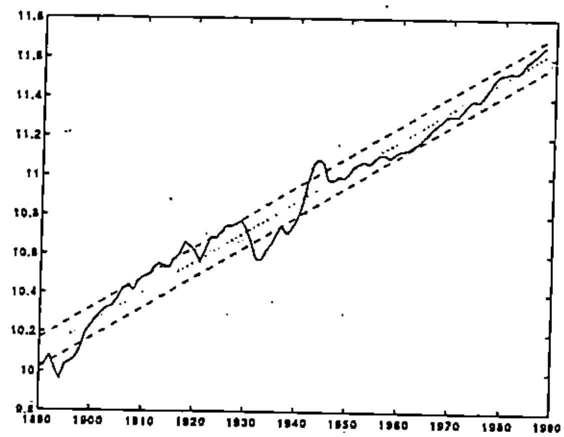


Figure 2.5 - Unemployment Rate

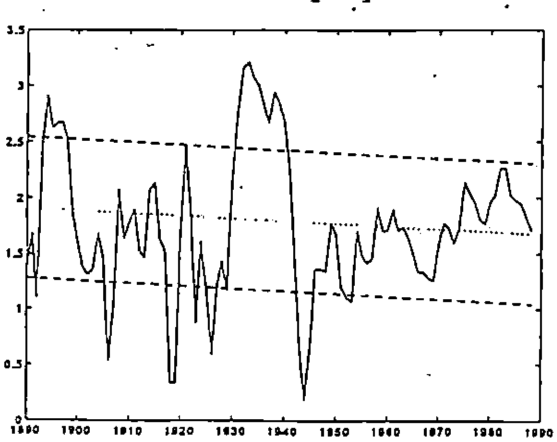
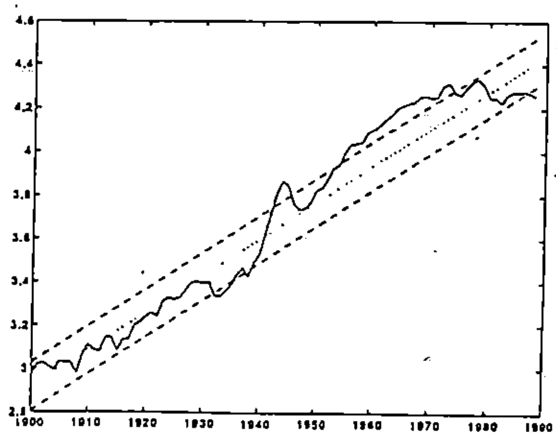


Figure 2.6 - Real Wages



Figures 2.1 - 2.14 (Continued)

Figure 2.7 - Nominal GNP

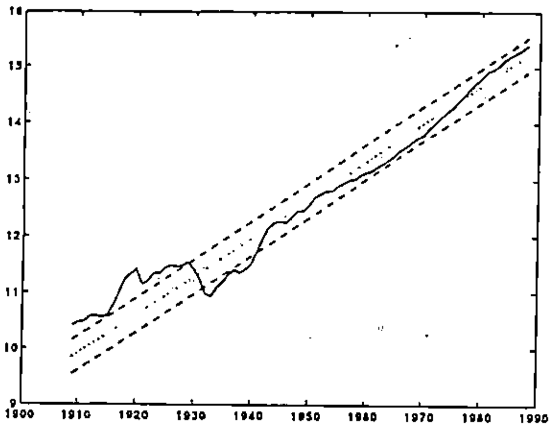


Figure 2.8 - Nominal Wages

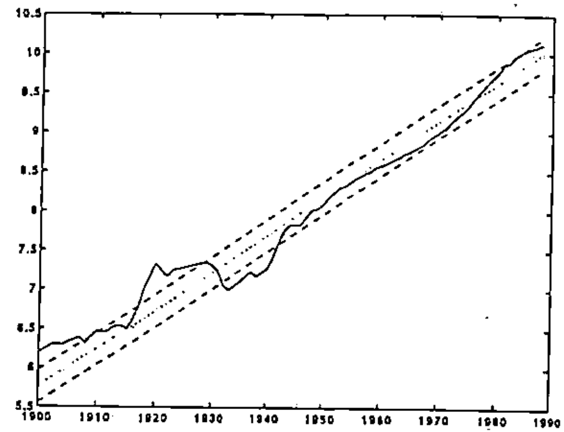


Figure 2.9 - Nominal Money Supply

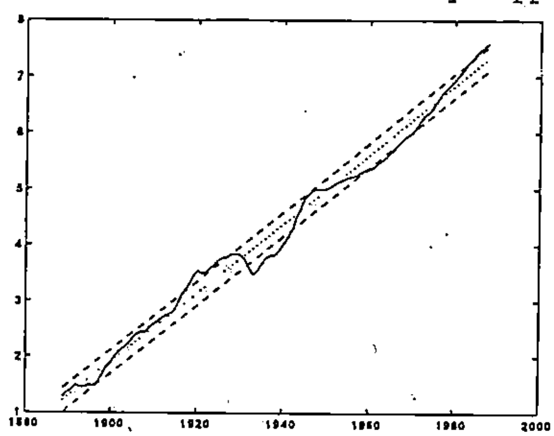


Figure 2.10 - Nominal Interest Rate

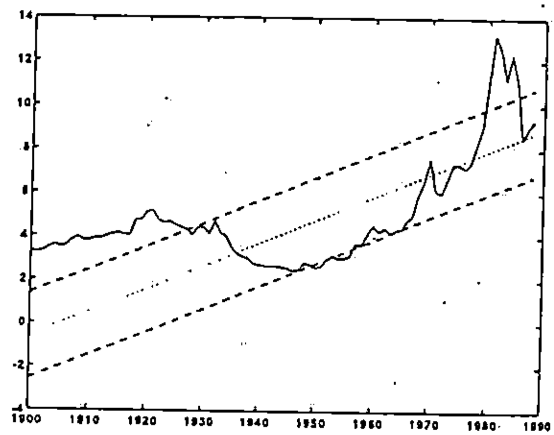


Figure 2.11 - S & P 500 Index

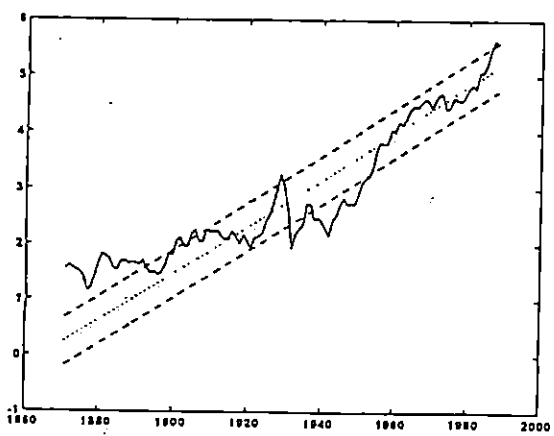
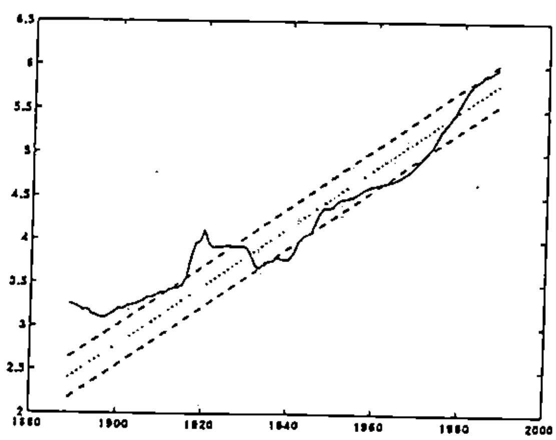


Figure 2.12 - GNP Deflator



Figures 2.1 - 2.14 (Continued)

Figure 2.13 - CPI

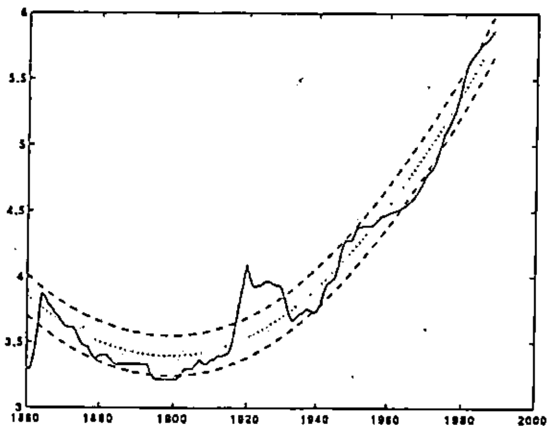
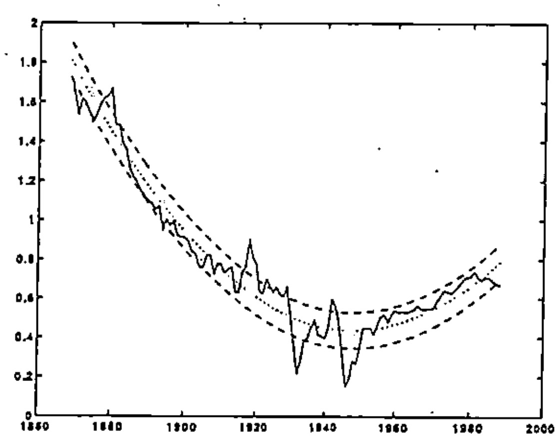


Figure 2.14 - Velocity



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